

JEE-Main-29-01-2024 (Memory Based) [EVENING SHIFT]

Maths

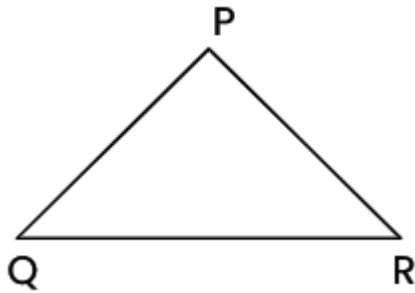
Question: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1 - \sin 2x} \, dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3}$

Find $\alpha - 3\beta$; find $\alpha\beta\gamma$

Solution:

$$\begin{aligned} & \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1 - \sin 2x} \, dx \\ &= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} |\cos x - \sin x| \, dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} |(\sin x - \cos x)| \, dx \\ &= +\sin x + \cos x \Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} + -\cos x - \sin x \Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}} \\ &= \left(\frac{1}{\sqrt{2}} - \frac{1}{2} + \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \right) + \left(\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} - \frac{1}{2} \right) \\ &= 2\sqrt{2} - \sqrt{3} - 1 \end{aligned}$$

Question: If P(3, 2, 3) Q(4, 6, 2) R(7, 3, 2) are the vertices of ΔPQR , then find $\angle QPR =$



Options:

- (a) $\cos^{-1} \frac{1}{18}$
- (b) $\frac{\pi}{6}$
- (c) $\frac{\pi}{3}$
- (d) $\cos^{-1} \frac{7}{18}$

Solution:



$$\overline{PQ} = (1, 4, -1)$$

$$\overline{PR} = (4, 1, -1)$$

$$\cos x = \frac{4 + 4 + 1}{18}$$

$$x = \frac{\pi}{3}$$

Question: Area bounded by $0 \leq y \leq \min\{x^2 + 2, 2x + 2\}$, $x \in [0, 3]$, then $12A$ is

Solution:

$$\int_0^2 x^2 + 2 + \int_2^3 2x + 2 - \frac{x^3}{3} + 2x$$

$$\left| 0^2 + x^2 + 2x \right| 2^3 \frac{8}{3} + 4 + 9 + 6 - (4 + 4)$$

$$A = \frac{41}{2}$$

$$12A = 164$$

Question: The remainder when $64^{32^{32}}$ is divided by 9 is _____.

Solution:

$$64^{32^{32}} \rightarrow 9$$

$$64 = 1 \pmod{9}$$

$$64^{32} = 1 \pmod{9}$$

$$64^{32^{32}} = 1 \pmod{9}$$

Question: $\cos(2 \sin^{-1} x) = \frac{1}{9}$ holds for $x = \frac{m}{n}$ and $\alpha\beta$ are the roots of the equation $mx^2 - nx -$

$m + n = 0$ ($\alpha > \beta$) then α, β lies on the line

Options:

(a) $5x - 8y = 9$

(b) $5x + 8y = 9$

(c) $8x + 5y = 9$

(d) $8x - 5y = 9$

Solution:



$$\cos(\sin^{-1} x) = \frac{1}{9}$$

$$1 - 2 \sin^2(\sin^{-1} x) = \frac{1}{9}$$

$$2x^2 = \frac{8}{9} \rightarrow x = \pm \frac{2}{3} = -\frac{2}{3}$$

$$\frac{2}{3} : 2x^2 - 3x + 1 = 9$$

$$\alpha = 1, \beta = \frac{1}{2}$$

$$5x + 8y = 9$$

Question: If α & β are roots of the equation $x^2 - \sqrt{6}x + 3$ and where $\text{Im}(\beta) < 0$ if

$$\frac{\alpha^{99}}{\beta} + \beta^{98} = 3^n (a + ib) \text{ then find a, b, n.}$$

Solution:

$$x^2 - \sqrt{6}x + 3 = 0$$

$$\beta, \alpha = \frac{\sqrt{6} \pm \sqrt{6}}{2} = \frac{\sqrt{6}(1 \pm i)}{\sqrt{2}\sqrt{2}} = \sqrt{3}e^{\pm i\frac{\pi}{4}}$$

$$\alpha = \sqrt{3}e^{i\frac{\pi}{4}}; \frac{\pi}{\beta} = i\alpha\beta = 3$$

$$\frac{\alpha^{99}}{\beta} + \beta^{98} = \alpha^{98}i + \beta^{98}$$

$$= 3^{49}[i \cdot i + (-j)]3^{49}(-1 - j)$$

$$n = 49, a = -1, B = -1$$

$$\text{Other method } \frac{\alpha^{99} + \beta^{99}}{\beta} = \frac{\alpha^{100}}{3} + \beta^{98} = 3^{49}(-1) + 3^{49}(-i)$$

Question: Distance of (2,4) from the line $2x + y + 2 = 0$ measured parallel to the line

$$\sqrt{3}x + y + 2 = 0:$$

Solution:

$$\left(2 - \frac{\sqrt{3}r}{2}, 4 + \frac{r}{2}\right)$$

$$2\left(2 - \frac{\sqrt{3}r}{2}\right) + 4 + \frac{r}{2} + 2 = 0$$

$$10 - \sqrt{3}r + \frac{r}{2} = 0$$

$$10 = \left(\sqrt{3} - \frac{1}{2}\right)r$$

$$r = \frac{20}{2\sqrt{3} - 1}$$



Question: a_1, a_2, \dots are in G.P. such that a,

$$= \frac{1}{8}, a_1 \neq a_2$$

and every term is equal to arithmetic mean of its two successive terms. Find $S_{20} - S_{18}$.

Solution:

$$a_1 = \frac{a_2 + a_3}{2}$$

$$a = \frac{ar + ar^2}{2}$$

$$r^2 + r = 2$$

$$r = -2$$

$$S_{20} - S_{18}$$

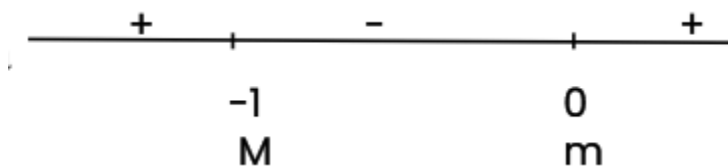
$$\frac{a[r^{20} - 1] - a[r^{10} - 1]}{r - 1}$$

$$\frac{a}{r - 1} [r^{20} - r^{18}]$$

$$\frac{1}{8(-3)} [2^{20} - 2^{18}]$$

$$\frac{2^{15}}{-3} [3] = -2^{15}$$

Question: $f(x) = 2x + 3(x)^{\frac{2}{3}}$. $x \in \mathbb{R}$.



Options:

- (a) it has one maxima no minima
- (b) it has one minima no maxima
- (c) it has 2 maxima and 1 minima
- (d) it has 1 maxima and 1 minima

Solution:

$$f(x) = 2x + 3(x)^{\frac{2}{3}}$$

$$f'(x) = 2 + 2x^{-1/3} = 0 = 2 \frac{(x^{1/3} + 1)}{x^{1/3}}$$

$$x = -1$$

$$x = -1 \text{ Maxima}$$

$$x = 0 \text{ Minima}$$

Question: $R : \{1, 2, 3, 4\} \rightarrow \{1, 2, 3, 4\}$ if $(1, 4), (1, 2) \in R$

Find the number of possible equivalence relations.

Solution: $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 4), (4, 1), (1, 2), (2, 1), (4, 2), (2, 4)\}$

Other $\{(1, 2), (3, 1), (2, 3), (3, 2), (4, 3), (3, 4)\}$

Either they all will come on not so 2.

Question: There are 8 identical book and 4 identical shelves Find the number of ways to arrange the book such that any shelf may be empty, every shelf can accommodate all the books

Options:

(a) 13

(b) 14

(c) 15

(d) 16

Solution:

$(8, 0, 0, 0)$ $(5, 3, 0, 0)$ $(4, 3, 1, 0)$

$(7, 1, 0, 0)$ $(5, 2, 1, 0)$ $(4, 2, 2, 0)$

$(6, 2, 0, 0)$ $(5, 2, 1, 0)$ $(4, 2, 2, 0)$

$(6, 1, 1, 0)$ $(4, 4, 0, 0)$ $(3, 3, 1, 1)$

$(2, 2, 2, 2)$ $(3, 3, 2, 0)$ $(3, 2, 2, 1)$

Total 15 ways.

Question: Find the probability that number selected from 1 to 50 such that number is divisible by at least one out of 4, 6 or by 7

Options:

(a) $\frac{21}{25}$

(b) $\frac{18}{50}$

(c) $\frac{8}{25}$

(d) $\frac{21}{25}$

Solution: $n(A \cup B \cup C)$

$= 12 + 8 + 7 - 4 - 1 - 1 + 0$

$= 21$

Probability = $\frac{21}{25}$

Question: A.B point of L_1 , & L_2 of shortest distance points from origin OA.OB